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Dissipative Phase Fluctuations In A Superconductor In Proximity To An Electron Gas

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Abstract

We study a two-dimensional superconductor in close proximity to a two-dimensional metallic sheet. The electrons in the superconducting sheet are coupled to those in the metallic sheet by the Coulomb interaction only. We obtain an effective phase-only action for the superconductor by integrating out all the electronic degrees of freedom in the problem. The Coulomb drag of the normal electrons in the metallic sheet is found to make the spectrum of phase-fluctuations in the superconductor, dissipative at long wavelengths. The dissipative co-efficient η is shown to be simply related to the normal state conductivities of the superconducting layer (σ_S)and the metallic sheet (σ_E) by the relation $\eta \propto \frac{\sigma_S \sigma_E}{\sigma_S + \sigma_E}$.

The study of phase fluctuations in disordered superconductors and Josephson junction arrays has received a great deal of attention [1,2,3] over the years as a mechanism for the superconductor-insulator transition [4,5] in systems with mesoscale disorder. The inhomogenities in these systems exist on a length scale much larger than the superconducting coherence length. The pairing mechanism is therefore believed to be relatively unaffected and the superconductor-insulator transition seen at low temperatures with increasing disorder is believed to be caused by enhanced fluctuations in the phase of the superconducting order parameter. A striking feature of this transition is the connection between an increasing normal state resistance and enhanced phase fluctuations in the superconductor which eventually destroy global phase coherence, thus leading to an insulating state. The discovery of a pseudo-gap phase [6,7,8] in the underdoped cuprates has given a renewed impetus to this field because of the possibility [9,10] that this phase corresponds to a collection of Cooper pairs that have lost global phase coherence.

Recently a superconductor-insulator transition was seen by A. J. Rimberg et. al. [11] in a two-dimensional array of Josephson junctions held within 100 nm of a two-dimensional electron gas (2DEG). A novel aspect of their work was the fact that the transition was achieved by tuning the conductivity of the 2DEG without altering the Josephson junction array directly. Motivated by this experiment we study, in this paper, the phase fluctuations in a 2-D superconductor held in close proximity (at a distance d) from a 2DEG. We model the superconductor by a two-dimensional BCS model together with a random potential and the 2DEG as an electron gas with a random potential. We explicitly include the full long-range Coulomb interaction, both inplane and interplanar. The Coulomb drag caused by the 2DEG is shown to make the long wavelength phase fluctuations in the superconductor dissipative in nature.

The dynamics of the system of electrons is described by the action

$$S = \int_0^\beta d\tau \int d^2x \int dz [L^{sc} + L^{eg} + L^{ef}] \tag{1a}$$

where

$$L^{sc} = \delta(z-d)[\sum_{\sigma} \bar{\psi}_{\sigma}(\mathbf{x},\tau)(\frac{\partial}{\partial\tau}+h_s)\psi_{\sigma}+\frac{|\Delta(\mathbf{x},\tau)|^2}{g}+(\Delta(\mathbf{x},\tau)\bar{\psi}_{\uparrow}(\mathbf{x},\tau)\bar{\psi}_{\downarrow}(\mathbf{x},\tau)+h.c.)] \quad (1b),$$

$$L^{eg} = \delta(z)[\sum_{\sigma} \bar{\chi}_{\sigma}(\mathbf{x},\tau)(\frac{\partial}{\partial\tau}+h_e)\chi_{\sigma}(\mathbf{x},\tau)] \quad (1c),$$

and

$$L^{ef} = \frac{(\nabla A_0(\mathbf{x},z,\tau))^2}{8\pi} \quad (1d).$$

The electrons at $(\mathbf{x},\tau)$ with spin σ are represented by the Grassman field variables $\bar{\psi}_{\sigma}(\mathbf{x},\tau)$, $\psi_{\sigma}(\mathbf{x},\tau)$ and $\bar{\chi}_{\sigma}(\mathbf{x},\tau)$, $\chi_{\sigma}(\mathbf{x},\tau)$ in the superconducting layer (at $z=d$) and the 2DEG (at $z=0$) respectively. Here

$$h_s = \frac{-\hbar^2\nabla^2}{2m_1} - ieA_0(\mathbf{x},d,\tau) + V_s(\mathbf{x}) - \epsilon_F^s \quad (2a)$$

and

$$h_e = \frac{-\hbar^2\nabla^2}{2m_2} - ieA_0(\mathbf{x},0,\tau) + V_e(\mathbf{x}) - \epsilon_F^e \quad (2b).$$

Thus, L^{sc} includes the electronic kinetic energy and the coupling of the superconducting electrons at $z=d$ to the Coulomb potential (A_0) as well as a random potential (V_s). The field Δ is the auxilliary Hubbard-Stratonovich field obtained from the BCS contact interaction and g is the strength of the attractive interaction. L^{eg} describes the 2DEG at $z=0$ together with its coupling to a random potential (V_e) and the Coulomb potential. L^{ef} gives the electric field energy of the system. We do not consider transverse vortex-like fluctuations and so the vector potential can be set equal to zero.

At low temperatures we can ignore fluctuations in the amplitude of the superconducting order parameter and make the replacement[12,13] $\Delta(\mathbf{x},\tau) = \Delta_0 \exp[i\theta(\mathbf{x},\tau)]$ where Δ_0 is the mean field value of $|\Delta(\mathbf{x},\tau)|$. Then on going over to a gauge in which the order parameter is real [13], (i.e. making the transformation $\psi_{\sigma}(\mathbf{x},\tau) \rightarrow \exp[\frac{i\theta}{2}]\psi_{\sigma}(\mathbf{x},\tau)$) we find that L^{sc} becomes

$$L^{sc} = \delta(z-d)[L^1 + L^2] \quad (3a)$$

where

$$L^1 = \sum_\sigma \bar{\psi}_\sigma[\frac{\partial}{\partial\tau} + \frac{i}{2}(\dot{\theta} - 2eA_0) + \frac{(\frac{\hbar\nabla}{i} + \frac{\hbar\nabla\theta}{2})^2}{2m_1} + V_s(\mathbf{x}) - \epsilon_F^s]\psi_\sigma \qquad (3b)$$

and

$$L^2 = \frac{\Delta_0^2}{g} + \Delta_0(\bar{\psi}_\uparrow\bar{\psi}_\downarrow + h.c.) \qquad (3c)$$

We now proceed to integrate out the fermions. We first consider L^{eg}. Then we have

$$\int D\bar{\chi}D\chi e^{-S^{eg}} = e^{-S^{eg}_{eff}[A_0]} \qquad (4a)$$

where

$$S^{eg} = \int_0^\beta d\tau \int d^3r L^{eg} \qquad (4b)$$

. We then compute $S^{eg}_{eff}[A_0]$ to second order in A_0 by performing a functional Taylor expansion of S^{eg}_{eff}. There is no contribution to S^{eg}_{eff} at first order in A_0 as the contribution coming from the electrons is exactly cancelled by the positive ionic background whose conribution has not been explicitly written earlier. It contributes a term of the type $ieA_0(\mathbf{x}, 0, \tau)\bar{n}_{eg}$ where $\bar{n}_{eg}$ is the average electronic density in the 2DEG.

At second order we find that S_{eff} is given by

$$S^{eg}_{eff} = \frac{e^2}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \int d^2x \int d^2x' A_0(\mathbf{x}, 0, \tau)A_0(\mathbf{x}', 0, \tau')P^0_{eg}(\mathbf{x} - \mathbf{x}', \tau - \tau') \qquad (5).$$

Here

$$P^0_{eg}(\mathbf{x} - \mathbf{x}', \tau - \tau') = \langle T_\tau[n_{eg}(\mathbf{x}, \tau)n_{eg}(\mathbf{x}', \tau')]\rangle \qquad (6),$$

$n_{eg}(\mathbf{x}, \tau)$ is the electron density fluctuation operator and averages are performed with respect to S^{eg} after setting A_0 to be zero.

We now integrate out the superconducting electrons. Once again we write

$$\int D\bar{\psi}D\psi e^{-S^{sc}} = e^{-S^{sc}_{eff}[\theta, A_0]} \qquad (7).$$

Proceeding as before, we find at first order there is no contribution to S^{sc}_{eff}. At second order we find

$$S^{sc}_{eff} = \frac{1}{8}\int_0^\beta d\tau d\tau' \int d^2x d^2x' [P^0_{sc}(\mathbf{x}-\mathbf{x}', \tau-\tau')f(\mathbf{x}, \tau)f(\mathbf{x}', \tau') + D(\mathbf{x}-\mathbf{x}', \tau-\tau')\nabla\theta(\mathbf{x}, \tau)\cdot\nabla'\theta(\mathbf{x}', \tau')] \qquad (8).$$

Here

$$f(\mathbf{x},\tau) = \dot{\theta}(\mathbf{x},\tau) - 2eA_0(\mathbf{x},d,\tau) \qquad (9a),$$

$$P^0_{sc}(\mathbf{x}-\mathbf{x}',\tau-\tau') = \langle T_\tau[n_{sc}(\mathbf{x},\tau)n_{sc}(\mathbf{x}',\tau')]\rangle \qquad (9b),$$

and

$$D(\mathbf{x}-\mathbf{x}',\tau-\tau') = \frac{\bar{n}_{sc}}{m_1}\delta^2(\mathbf{x}-\mathbf{x}')\delta(\tau-\tau') - \frac{1}{m_1^2}\langle T_\tau[p_{x_1}(\mathbf{x},\tau)p_{x_1}(\mathbf{x}',\tau')]\rangle \qquad (9c).$$

n_{sc} represents the electron density fluctuation operator in the superconducting state and p_{x_1} is the momentum density operator along the x_1 direction. All averages are performed with respect to L_{sc} with A_0 and θ set equal to zero.

Including the electric field energy in L^{ef} we find that the effective action for the system becomes

$$S_{eff} = S^{eg}_{eff} + S^{sc}_{eff} + S^{ef} \qquad (10a)$$

where S^{eg}_{eff} and S^{sc}_{eff} are defined in Eqs. (5) and (8) respectively and

$$S^{ef} = \int_0^\beta d\tau \frac{[\nabla A_0(\mathbf{x},z,\tau)]^2}{8\pi} \qquad (10b)$$

Varying the effective action S_{eff} with respect to $A_0(\mathbf{x},z,\tau)$ we obtain the equation of motion to be

$$\frac{\nabla^2 A_0}{4\pi} = \delta(z-d)X(\mathbf{x},\tau) + \delta(z)Y(\mathbf{x},\tau) \qquad (11a)$$

where

$$X(\mathbf{x},\tau) = \frac{-e}{2}\int_0^\beta d\tau' \int d^2x' P^0_{sc}(\mathbf{x}-\mathbf{x}',\tau-\tau')(\frac{\partial\theta}{\partial\tau'} - 2eA_0(\mathbf{x}',d,\tau')) \qquad (11b)$$

and

$$Y(\mathbf{x},\tau) = e^2\int_0^\beta d\tau' \int d^2x' P^0_{eg}(\mathbf{x}-\mathbf{x}',\tau-\tau')A_0(\mathbf{x}',0,\tau') \qquad (11c)$$

Going over to Fourier space by using the transformations

$$\theta(\mathbf{x},\tau) = \frac{1}{\beta}\sum_{\nu_m}\int \frac{d^2q}{(2\pi)^2}\exp i[\mathbf{q}\cdot\mathbf{x} - \nu_m\tau]\theta(\mathbf{q},\nu_m) \qquad (12a)$$

and

$$A_0(\mathbf{x},z,\tau) = \frac{1}{\beta}\sum_{\nu_m}\int \frac{d^2q}{(2\pi)^2}\int\frac{dk}{2\pi}\exp i[\mathbf{q}\cdot\mathbf{x} + kz - \nu_m\tau]\tilde{A}_0(\mathbf{q},k,\nu_m) \qquad (12b)$$

we find that Eq. (11) can be rewritten as

$$\tilde{A}_0(\mathbf{q},k,\nu_m)=\frac{1}{q^2+k^2}[-2\pi e\exp[-ikd]X_1(\mathbf{q},\nu_m)-4\pi e^2\exp[-ikd]X_2(\mathbf{q},\nu_m)-4\pi e^2X_3(\mathbf{q},\nu_m)] \qquad (13a).$$

Here

$$X_1(\mathbf{q},\nu_m)=P^0_{sc}(\mathbf{q},\nu_m)i\nu_m\theta(\mathbf{q},\nu_m) \qquad (13b),$$

$$X_2(\mathbf{q},\nu_m)=P^0_{sc}(\mathbf{q},\nu_m)A_0(\mathbf{q},z=d,\nu_m) \qquad (13c),$$

and

$$X_3(\mathbf{q},\nu_m)=P^0_{eg}(\mathbf{q},\nu_m)A_0(\mathbf{q},z=0,\nu_m) \qquad (13d).$$

Eq. (13) is an integral equation for $\tilde{A}_0(\mathbf{q},k,\nu_m)$. Solving for $A_0(\mathbf{q},d,\nu_m)$ and $A_0(\mathbf{q},0,\nu_m)$ we find, after some algebra, the relations

$$A_0(\mathbf{q},d,\nu_m)[1+\frac{2\pi e^2}{q}P^0_{sc}(\mathbf{q},\nu_m)]+\frac{2\pi e^2}{q}e^{-qd}P^0_{eg}(\mathbf{q},\nu_m)A_0(\mathbf{q},0,\nu_m)=\frac{-\pi ei\nu_m}{q}P^0_{sc}(\mathbf{q},\nu_m)\theta(\mathbf{q},\nu_m) \qquad (14a)$$

and

$$A_0(\mathbf{q},0,\nu_m)[1+\frac{2\pi e^2}{q}P^0_{eg}(\mathbf{q},\nu_m)]+\frac{2\pi e^2}{q}e^{-qd}P^0_{sc}(\mathbf{q},\nu_m)A_0(\mathbf{q},d,\nu_m)=\frac{-\pi ei\nu_m}{q}e^{-qd}P^0_{sc}(\mathbf{q},\nu_m)\theta(\mathbf{q},\nu_m) \qquad (14b).$$

Solving Eqs. (14a) and (14b) for $A_0(\mathbf{q},d,\nu_m)$ we find that

$$A_0(\mathbf{q},d,\nu_m)=\frac{2\pi ei\nu_m\theta(\mathbf{q},\nu_m)R_1}{1+R_2} \qquad (15a)$$

where

$$R_1=\frac{-P^0_{sc}(\mathbf{q},\nu_m)}{2q}-\frac{\pi e^2}{q^2}P^0_{sc}(\mathbf{q},\nu_m)P^0_{eg}(\mathbf{q},\nu_m)(1-e^{-2qd}) \qquad (15b)$$

and

$$R_2=\frac{2\pi e^2}{q}(P^0_{sc}(\mathbf{q},\nu_m)+P^0_{eg}(\mathbf{q},\nu_m))+\frac{4\pi^2e^4}{q^2}P^0_{sc}(\mathbf{q},\nu_m)P^0_{eg}(\mathbf{q},\nu_m)(1-e^{-2qd}) \qquad (15c).$$

Substituting the equation of motion (Eq. (11)) in the expression for S_{eff} (Eq. (10)) we find that S_{eff} becomes

$$S_{eff}=S^1_{eff}+S^2_{eff} \qquad (16a)$$

where

$$S^1_{eff} = \frac{1}{8}\int_0^\beta d\tau d\tau' \int d^2x d^2x' P^0_{sc}(\mathbf{x}-\mathbf{x}', \tau-\tau')\frac{\partial\theta}{\partial\tau} f(\mathbf{x}', \tau') \qquad (16b)$$

and

$$S^2_{eff} = \frac{1}{8}\int_0^\beta d\tau d\tau' \int d^2x d^2x' D(\mathbf{x}-\mathbf{x}', \tau-\tau')\nabla\theta(\mathbf{x}, \tau)\cdot\nabla'\theta(\mathbf{x}', \tau') \qquad (16c).$$

For simplicity, we first consider the limits $d \to \infty$ and $d \to 0$. In the former case we find that

$$A_0(\mathbf{q}, d, \nu_m) = \frac{-(\pi e/q)P^0_{sc}(\mathbf{q}, \nu_m)i\nu_m\theta(\mathbf{q}, \nu_m)}{1 + (2\pi e^2/q)P^0_{sc}(\mathbf{q}, \nu_m)} \qquad (17).$$

Notice that in this limit the layers become decoupled and P^0_{eg} drops out of the expression for A_0 in the superconducting layer. Substituting Eq. (17) in the earlier relation (Eq. (16b)) for S^1_{eff} we get

$$S^1_{eff} = \frac{1}{8\beta}\sum_{\nu_m}\int \frac{d^2q}{(2\pi)^2}\frac{\nu_m^2 \mid \theta(\mathbf{q}, \nu_m)\mid^2 P^0_{sc}(\mathbf{q}, \nu_m)}{1 + (2\pi e^2/q)P^0_{sc}(\mathbf{q}, \nu_m)} \qquad (18)$$

This in combination with S^2_{eff} (Eq. (16c)) is the usual action [13] for phase fluctuations in a two-dimensional superconductor and at long wavelengths the phase fluctuations obey a dispersion relation proportional to $q^{1/2}$.

Assuming that the onset of superfluid order doesn't affect the electronic compressibility very much, we have the relation

$$1 + (2\pi e^2/q)P^0_{sc}(\mathbf{q}, \nu_m) \approx \epsilon_s(\mathbf{q}, \nu_m) \qquad (19)$$

where $\epsilon_s(\mathbf{q}, \nu_m)$ is the dielectric function of the superconductor in its normal state. This dielectric function, in turn, is related to the normal state conductivity (σ_S) of the superconductor by the relation

$$\epsilon_s(\mathbf{q}, \nu_m) = 1 + \frac{4\pi\hbar\sigma_S(\mathbf{q}, \nu_m)}{\mid \nu_m \mid} \approx \frac{4\pi\hbar\sigma_S(\mathbf{q}, \nu_m)}{\mid \nu_m \mid} \qquad (20a)$$

at low frequencies. In a similar manner, at low frequencies we have the relation

$$P^0_{sc}(\mathbf{q}, \nu_m) \approx \frac{4\pi\hbar\sigma_S(\mathbf{q}, \nu_m)}{(2\pi e^2/q)\mid \nu_m \mid} \qquad (20b).$$

Making these substitutions in Eq. (18) we find that S_{eff} reduces to

$$S_{eff} = \frac{1}{8\beta} \sum_{\nu_m} \int \frac{d^2q}{(2\pi)^2} \mid \theta(\mathbf{q}, \nu_m) \mid^2 [\frac{q\nu_m^2}{2\pi e^2} + D(\mathbf{q}, \nu_m) q^2] \tag{21}$$

which is the plasma action for a two-dimensional superconductor.

We now turn our attention to the other limit of $d \to 0$. In this case, we find that

$$A_0(\mathbf{q}, d, \nu_m) = \frac{-\pi(e/q) i\nu_m P^0_{sc}(\mathbf{q}, \nu_m)}{1 + 2\pi e^2/q(P^0_{sc}(\mathbf{q}, \nu_m) + P^0_{eg}(\mathbf{q}, \nu_m))} \qquad (22).$$

Using the result of Eq. (22) for A_0 and taking the low frequency limit as before we find that S_{eff} becomes

$$S_{eff} = \frac{1}{8\beta} \sum_{\nu_m} \int \frac{d^2q}{(2\pi)^2} \mid \theta(\mathbf{q}, \nu_m) \mid^2 [\frac{\hbar \sigma_S \sigma_E q \mid \nu_m \mid}{\sigma_S + \sigma_E} + D(\mathbf{q}, \nu_m) q^2] \tag{23}$$

where σ_E is the conductivity of the 2DEG and we have replaced the wavevector and frequency dependent conductivities by their d.c. values in the low energy limit. In writing Eq. (23) we have made use of relations similar to Eqs. (19), (20a) and (20b) for the electron gas as well.

It is clear from Eq. (23) that in this case the phase fluctuations are dissipative in nature and the corresponding viscosity co-efficient (η) is related to the conductivities of the two types of electrons by the relation

$$\eta \propto \frac{\sigma_S \sigma_E}{\sigma_S + \sigma_E} \qquad (24).$$

We will now consider the general expression for $A_0(\mathbf{q}, d, \nu_m)$ (Eq. (15)). In this case we find that S_{eff} becomes

$$S_{eff} = \frac{1}{8\beta} \sum_{\nu_m} \int \frac{d^2q}{(2\pi)^2} \mid \theta(\mathbf{q}, \nu_m) \mid^2 [\frac{Y_1(\mathbf{q}, \nu_m)}{Y_2(\mathbf{q}, \nu_m)} \nu_m^2 + D(\mathbf{q}, \nu_m) q^2] \tag{25a}$$

where

$$Y_1(\mathbf{q}, \nu_m) = P^0_{sc}(\mathbf{q}, \nu_m) \epsilon_{eg}(\mathbf{q}, \nu_m) \tag{25b}$$

and

$$Y_2(\mathbf{q}, \nu_m) = \epsilon_{eg}(\mathbf{q}, \nu_m) + \epsilon_{sc}(\mathbf{q}, \nu_m) - 1 + (2\pi e^2/q) P^0_{sc}(\mathbf{q}, \nu_m) P^0_{eg}(\mathbf{q}, \nu_m)(1 - e^{-2qd}) \qquad (25c).$$

Here $\epsilon_{eg}(\mathbf{q},\nu_m) = 1 + \frac{2\pi e^2}{q} P^0_{eg}(\mathbf{q},\nu_m)$ is the dielectric function of the 2DEG.

It is clear from Eq. (25) that d^{-1} defines a crossover scale and phase fluctuations with $q \gg d^{-1}$ ($e^{-qd} \rightarrow 0$) behave like the $d \rightarrow \infty$ limit considered earlier i.e. they are propogating modes. In this case $Z(\mathbf{q},\nu_m) = \frac{Y_1(\mathbf{q},\nu_m)}{Y_2(\mathbf{q},\nu_m)}$ reduces to $\frac{P^0_{sc}(\mathbf{q},\nu_m)}{1+\frac{2\pi e^2}{q}P^0_{sc}(\mathbf{q},\nu_m)}$. The action for these modes becomes independent of $P^0_{eg}(\mathbf{q},\nu_m)$ showing that the short wavelength modes are unaffected by the presence of the metallic layer. On further taking the low frequency limit, as before, $Z(\mathbf{q},\nu_m)$ becomes $\frac{q}{2\pi e^2}$ as in the special case of $d \rightarrow \infty$ considered earlier.

The opposite limit of $q \ll d^{-1}$ is like the case of $d \rightarrow 0$ treated earlier. In that case $e^{-2qd} \rightarrow 1$ and $Z(\mathbf{q},\nu_m)$ reduces to

$$Z(\mathbf{q},\nu_m) = \frac{P^0_{sc}(\mathbf{q},\nu_m)\epsilon_{eg}(\mathbf{q},\nu_m)}{\epsilon_{eg}(\mathbf{q},\nu_m) + \epsilon_{sc}(\mathbf{q},\nu_m) - 1} \qquad (26).$$

Now taking the low frequency limit we find that $Z(\mathbf{q},\nu_m)$ becomes

$$Z(\mathbf{q},\nu_m) = \frac{\hbar\sigma_S(\mathbf{q},\nu_m)\sigma_E(\mathbf{q},\nu_m)}{\mid \nu_m \mid [\sigma_S(\mathbf{q},\nu_m) + \sigma_E(\mathbf{q},\nu_m)]} \qquad (27).$$

Further in the low energy limit we can replace the conductivities $\sigma_S(\mathbf{q},\nu_m)$ and $\sigma_E(\mathbf{q},\nu_m)$ by their d.c. values σ_S and σ_E. Then it is clear from Eqs. (25) and (27) that the long wavelength modes are dissipative in nature. Their behaviour is identical with the $d \rightarrow 0$ limit considered earlier. Notice once again that the co-efficient of dissipation (η) is related simply to the normal state conductivities of the superconducting sheet (σ_S) and the metallic sheet (σ_E) by the simple relation $\eta \propto \frac{\sigma_E\sigma_S}{\sigma_E+\sigma_S}$. Thus the amount of dissipation can be tuned by changing the conductivity in either sheet.

Finally, let us summarise the main results of this paper. We have obtained the effective action for the phase fluctuations in a two-dimensional superconductor in close proximity to a metallic sheet. We find that the long wavelength ($q < d^{-1}$) modes are dissipative. The coefficient of dissipation is related to the normal state conductivities of both layers. The short wavelength ($q > d^{-1}$) modes are unaffected by the presence of the metallic sheet. The wavevector d^{-1} defines a crossover scale for the behaviour of the phase fluctuations to change from one regime (dissipative) to the other regime (propogating).

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